GCE214
Applied Mechanics-Statics

Lecture 03: 20/09/2017

Dr. Ayokunle O. Balogun
balogun.ayokunle@lmu.edu.ng

Class: Wednesday (3–5 pm)
Venue: LT1

Etiquettes and MOP

- Attendance is a requirement.
- There may be class assessments, during or after lecture.
- Computational software will be employed in solving problems
- Conceptual understanding will be tested
- Lively discussions are integral part of the lectures.

Lecture content

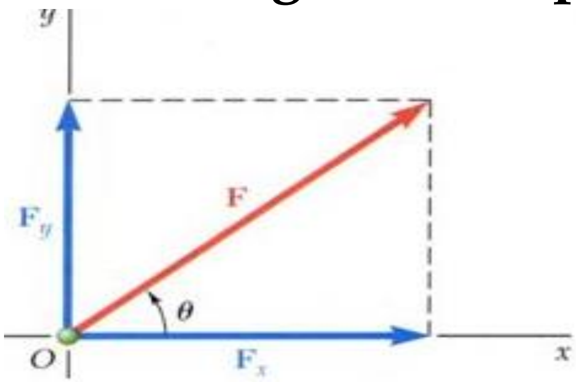
- Representation and Resolution of Vector of Forces in 2D
- Free-body Diagram
- Equilibrium of Forces
- Rectangular Components of Force in Space

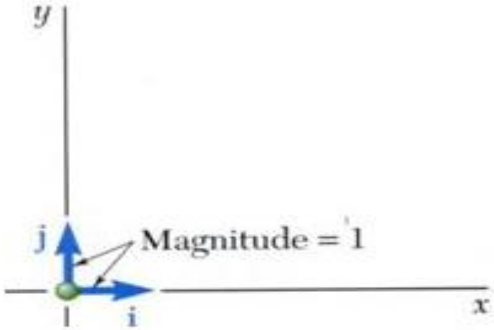
Recommended textbook

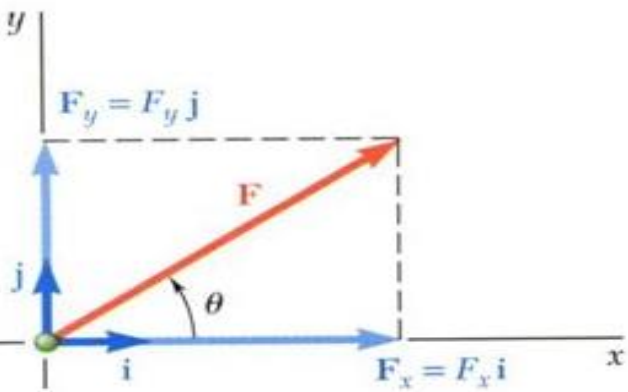
- Vector Mechanics for Engineers: Statics and Dynamics by Beer, Johnston, Mazurek, Cornwell. 10th Edition

VECTORS OF FORCES IN 2D

- **Rectangular Components of a Force: Unit Vectors**







- It is possible to resolve a force vector into perpendicular components so that the resulting parallelogram is a rectangle. $\boldsymbol{F}_x$ and $\boldsymbol{F}_y$ are referred to as ***rectangular vector components*** and

$$\boldsymbol{F} = \boldsymbol{F}_x + \boldsymbol{F}_y$$

- Define perpendicular ***unit vectors*** $\boldsymbol{i}$ and $\boldsymbol{j}$ which are parallel to the x and y axes.

- Vector components may be expressed as products of the unit vectors with the scalar magnitudes of the vector components.

$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j}$$

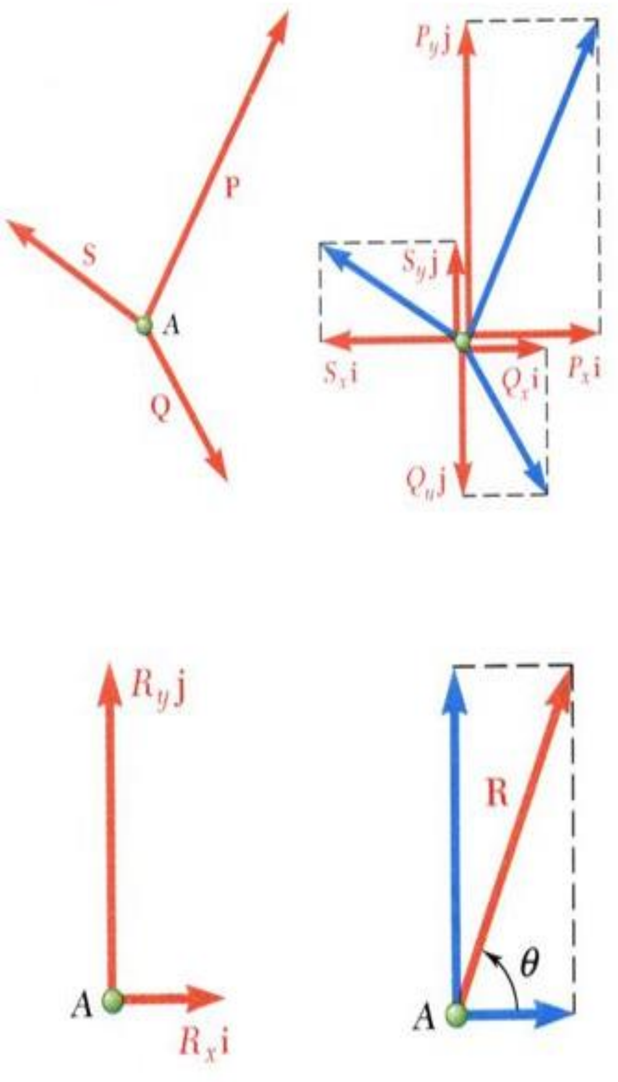
- F_x and F_y are referred to as ***scalar components*** of $\boldsymbol{F}$

VECTORS OF FORCES IN 2D

3. A force of 800 N is exerted on a bolt, A, 145° measured from the horizontal. Determine the horizontal and vertical components of the force.
4. A man pulls with a force of 300 N on a rope attached to a building. The man is positioned 8 m away horizontally and 6 m at the base of the building (vertically) from the point of attachment, A. What are the horizontal and vertical components of the force exerted by the rope at point A?
5. A force $\boldsymbol{F} = (318\,\boldsymbol{i} + 680\,\boldsymbol{j})$ N is applied to a bolt A. Determine the magnitude of the force and the angle α it forms with the horizontal.

VECTORS OF FORCES IN 2D

Addition of Forces by Summing Components



- The resultant of 3 or more concurrent forces is $\boldsymbol{R} = \boldsymbol{P} + \boldsymbol{Q} + \boldsymbol{S}$
- Resolve each force into the rectangular components

$$R_x\boldsymbol{i} + R_y\boldsymbol{j} = P_x\boldsymbol{i} + P_y\boldsymbol{j} + Q_x\boldsymbol{i} + Q_y\boldsymbol{j} + S_x\boldsymbol{i} + S_y\boldsymbol{j}$$
$$= (P_x + Q_x + S_x)\boldsymbol{i} + \left(P_y + Q_y + S_y\right)\boldsymbol{j}$$

- The scalar components of the resultant are equal to the sum of the corresponding scalar components of the given forces

$$R_x = P_x + Q_x + S_x \qquad R_y = P_y + Q_y + S_y$$
$$\sum F_x \qquad \sum F_y$$

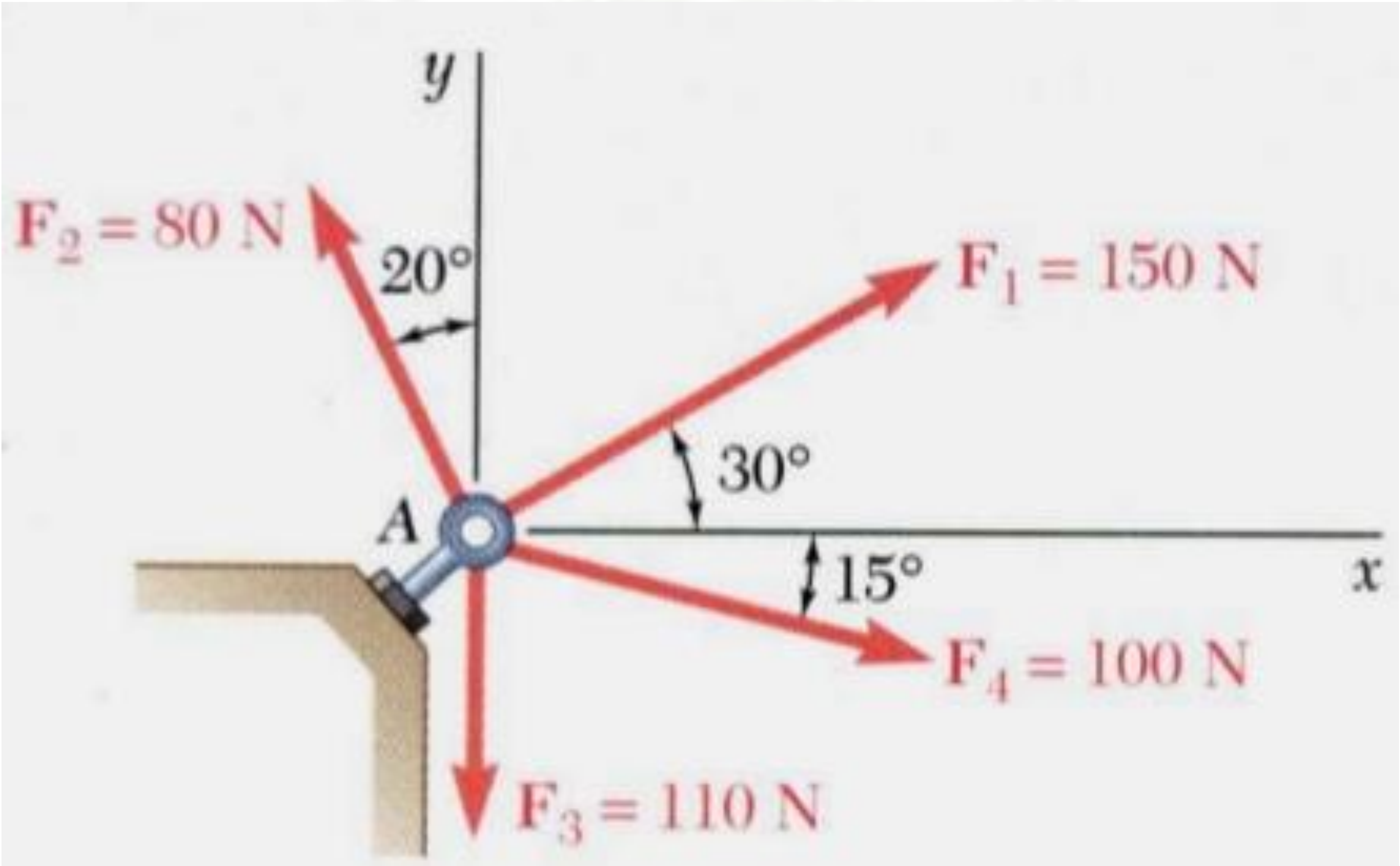
- To find the resultant magnitude and direction,

$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$

VECTORS OF FORCES IN 2D

EXAMPLES

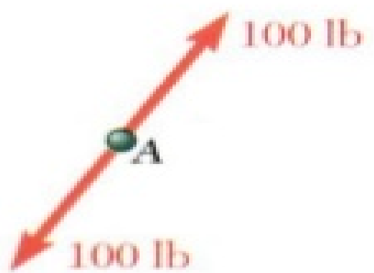
6. Four forces act on a bolt as shown in the Fig below. Determine the resultant of the forces on the bolt

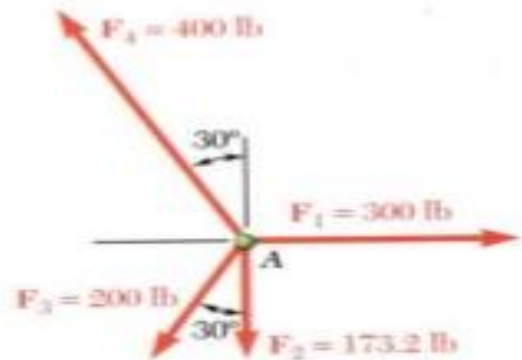


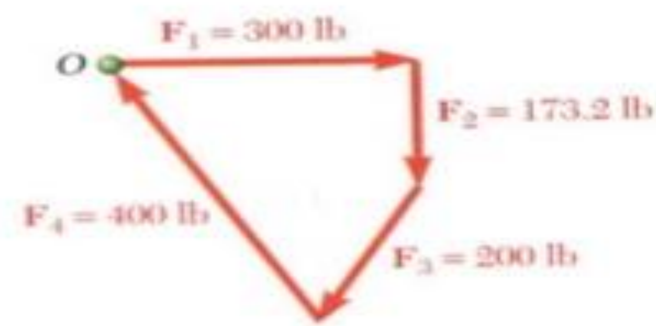
VECTORS OF FORCES IN 2D

Equilibrium of A Particle

- When the resultant of all the forces acting on a particle is zero, the particle is in ***equilibrium***
- Newton's first law: If the resultant forces acting on a particle is zero, the particle will remain at rest, or will move with constant speed in a straight line





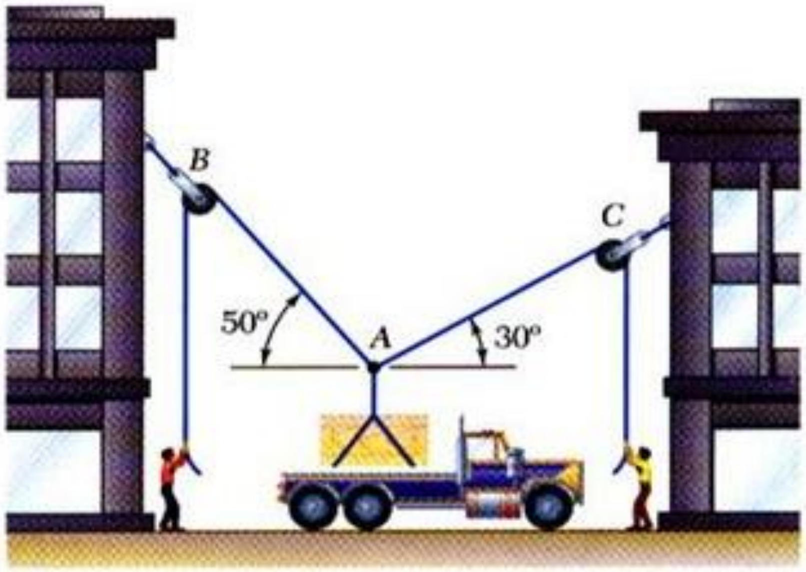


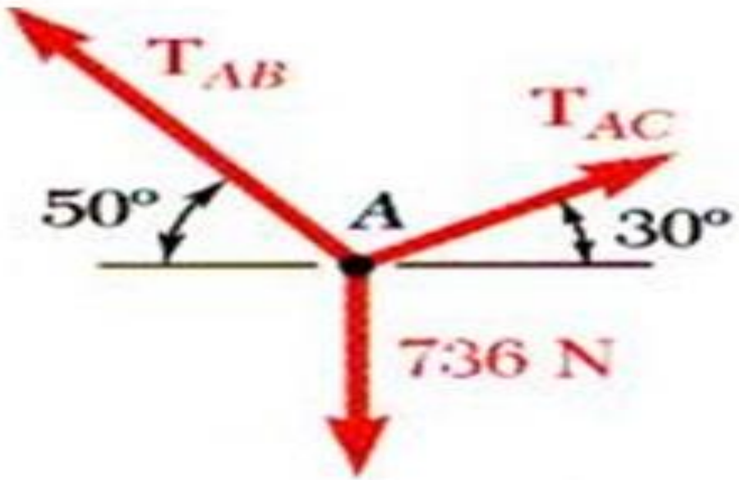
- Particles acted upon by two forces:
  - equal magnitude
  - same line of action
  - opposite direction
- Particle acted upon by three or more forces:
  - Graphical solution yields a closed polygon
  - algebraic solution

$$\boldsymbol{R} = \sum \boldsymbol{F} = 0$$

$$\sum F_x = 0 \qquad \sum F_y = 0$$

VECTORS OF FORCES IN 2D





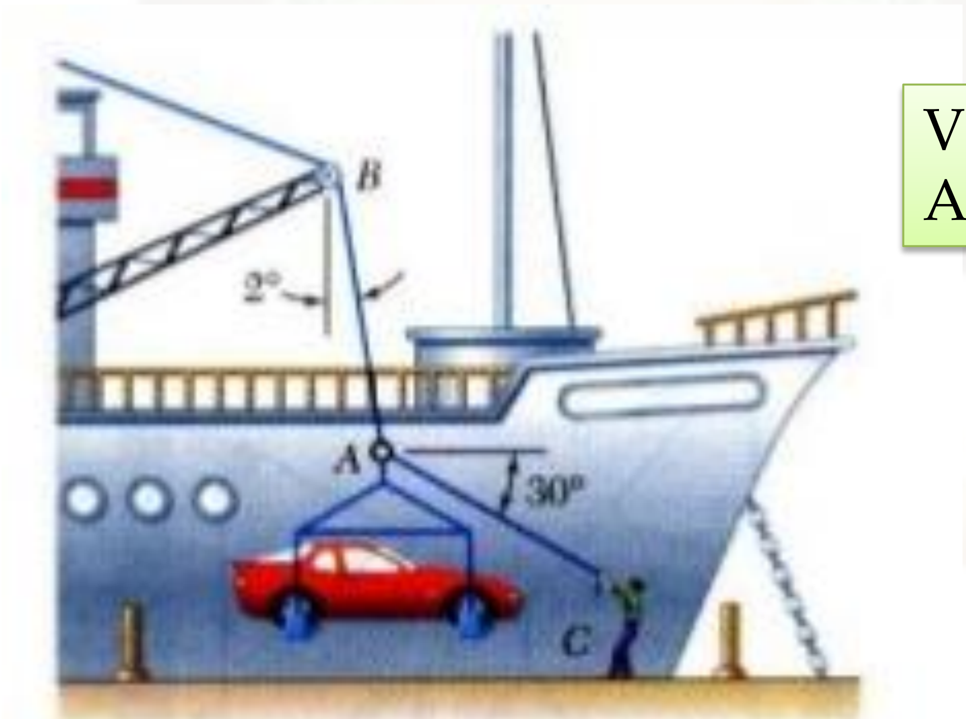
- *Space diagram:* A sketch the physical conditions of the problem, usually provided with the problem statement, or represented by the actual physical situation

- *Free Body Diagram:* A sketch showing only forces on the selected particle.
  <u>This must be created by you</u>

VECTORS OF FORCES IN 2D

EXAMPLES

7. In a ship-offloading operation, a 162 kg automobile is supported by a cable. A rope is tied to the cable at *A* and pulled in order to center the automobile over its intended position. The angle between the cable and the vertical is 2°, while the angle between the rope and the horizontal is 30°. What is the tension in the rope?

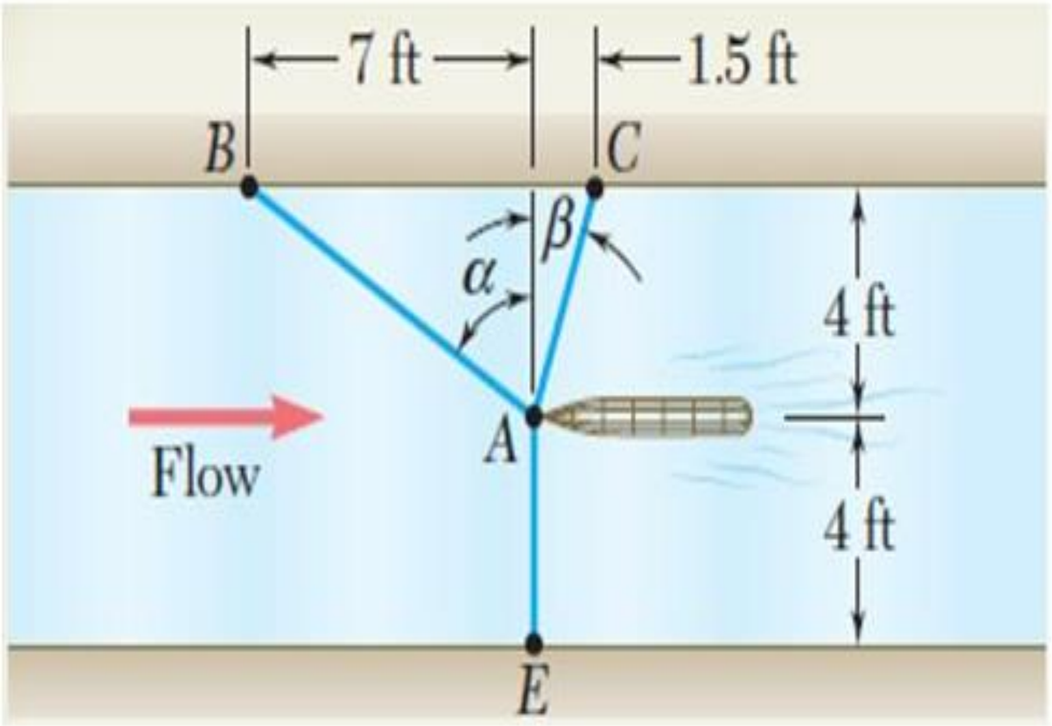


Verify that the particle A is in equilibrium

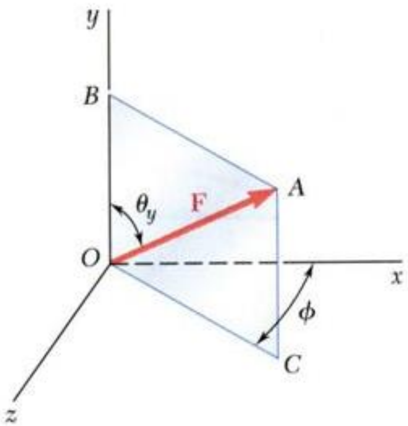
VECTORS OF FORCES IN 2D

EXAMPLES

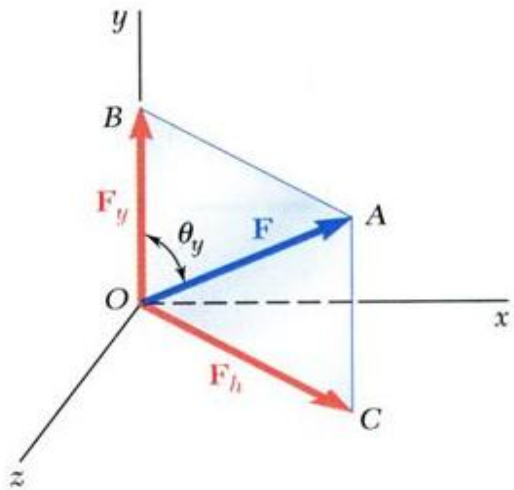
8. As part of the design of a new sailboat, it is desired to determine the drag force which may be expected at a given speed. To do so, a model of the proposed hull is placed in a test channel and three cables are used to keep its bow on the centerline of the channel. Dynamometer readings indicate that for a given speed, the tension is 18 N in cable *AB* and 27 N in cable *AE*. Determine the drag force exerted on the hull and the tension in cable *AC*.

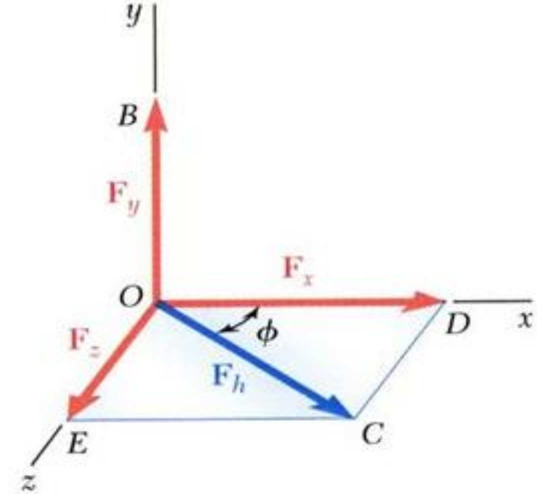


RECTANGULAR COMPONENTS IN SPACE



- The vector **F** is contained in the plane OBAC.





- Resolving $\boldsymbol{F_h}$ into rectangular compoents yield:

$$F_x = F_h \cos \emptyset = F \sin \theta_y \cos \emptyset$$
$$F_y = F_h \sin \emptyset = F \sin \theta_y \sin \emptyset$$

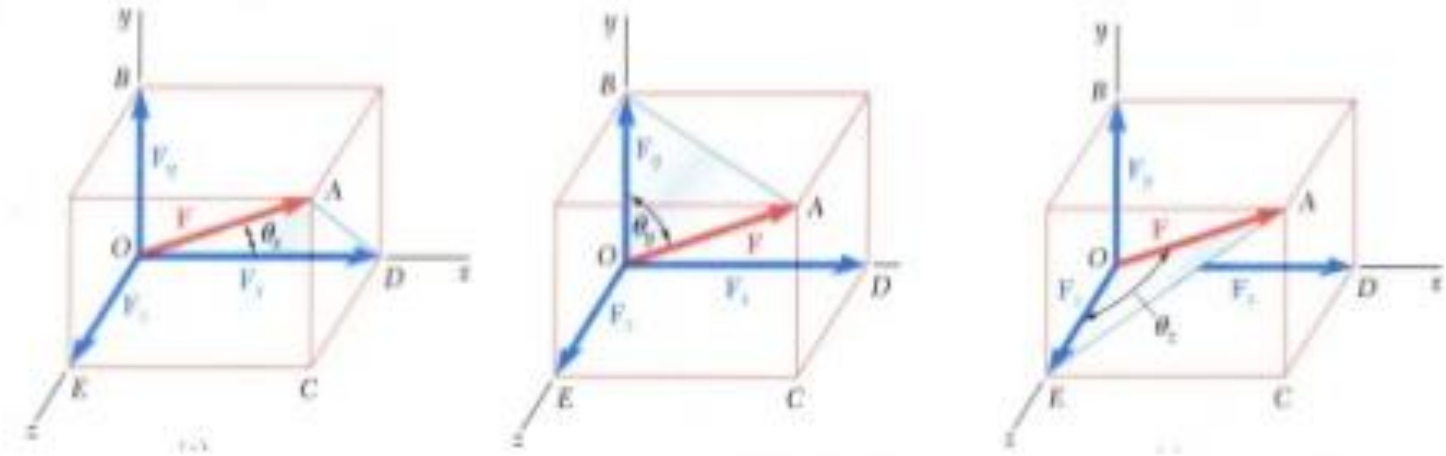
- Resolving **F** into horizontal and vertical components yield:

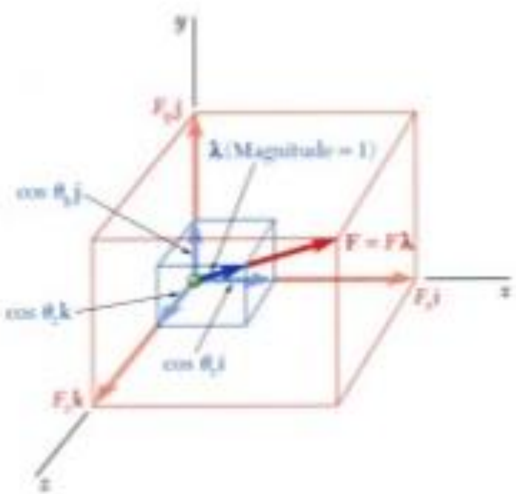
$$F_y = F \cos \theta_y$$
$$F_h = F \sin \theta_y$$

Applying Pythagorean theorem to triangle OAB and OCD and eliminating F_h yields

$$F = \sqrt{(F_x)^2 + \left(F_y\right)^2 + (F_z)^2}$$

RECTANGULAR COMPONENTS IN SPACE





- With the angles between $\boldsymbol{F}$ and the axes,

$$F_x = F\cos\theta_x, F_y = F\cos\theta_y, F_z = F\cos\theta_z$$

$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$$

$$= F(\cos\theta_x\boldsymbol{i} + \cos\theta_y\boldsymbol{j} + \cos\theta_z\boldsymbol{k})$$

$$= F\boldsymbol{\lambda}$$

Where

$$\boldsymbol{\lambda} = \cos\theta_x\boldsymbol{i} + \cos\theta_y\boldsymbol{j} + \cos\theta_z\boldsymbol{k}$$

- $\boldsymbol{\lambda}$ is a unit vector along the line of action of $\boldsymbol{F}$ and $\cos\theta_x$, $\cos\theta_y$, $\cos\theta_z$ are the direction cosines for $\boldsymbol{F}$

RECTANGULAR COMPONENTS IN SPACE

$$\boldsymbol{\lambda} = \cos\theta_x \boldsymbol{i} + \cos\theta_y \boldsymbol{j} + \cos\theta_z \boldsymbol{k}$$

- $\boldsymbol{\lambda}$ is a unit vector along the line of action of $\boldsymbol{F}$ and $\cos\theta_x$, $\cos\theta_y$, $\cos\theta_z$ are the direction cosines for $\boldsymbol{F}$
- The magnitude of the unit vector $\boldsymbol{\lambda}$ along the three axes are

$$I_x = \cos\theta_x,\ I_y = \cos\theta_y,\ I_z = \cos\theta_z$$

- Recall that the sum of the squares of components of a vector is equal to square of its magnitude $\therefore$

$$I_x^2 + I_y^2 + I_z^2 = 1$$

- From the equation directly above

$$cos^2\theta_x + cos^2\theta_y + cos^2\,\theta_z = 1$$

- Note that the values of the angles are not independent of each other
- Once F_x, F_y and F_z are known the magnitude F can be determined and the direction cosines as follows

$$\cos\theta_x = \frac{F_x}{F},\ \cos\theta_y = \frac{F_y}{F},\ \cos\theta_z = \frac{F_z}{F}$$

RECTANGULAR COMPONENTS IN SPACE

EXAMPLES

9. A force of 500 N forms an angle of 60°, 45° and 120° respectively with the x, y, and z axes. Find the components F_x, F_y and F_z of the force

10. A force $\boldsymbol{F}$ has the components $F_x = 9.1\ kg$, $F_y = -13.6\ kg$, $F_z = 27.2\ kg$. Determine its magnitude $\boldsymbol{F}$ and the angles θ_x, θ_y, and θ_z it forms with the coordinate axes

RECTANGULAR COMPONENTS IN SPACE

Addition of Concurrent Forces in Space

- The resultant of two or more forces in space is the sum of the respective rectangular components

$$\boldsymbol{R} = \sum \boldsymbol{F}$$

- Resolve each force into the rectangular components

$$R_x\boldsymbol{i} + R_y\boldsymbol{j} + R_k\boldsymbol{k} = \sum\left(F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}\right) = \left(\sum F_x\right)\boldsymbol{i} + \left(\sum F_y\right)\boldsymbol{j} + \left(\sum F_z\right)\boldsymbol{k}$$

- $\therefore$

$$R_x = \sum F_x\,, \qquad R_y = \sum F_y, \qquad R_z = \sum F_z$$

- To find the resultant magnitude and direction,

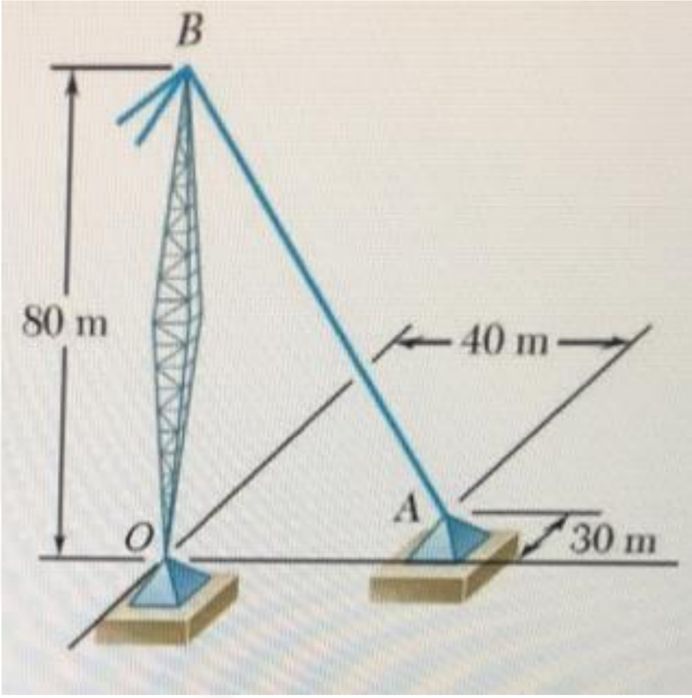
$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$\theta_x = \tan^{-1}\frac{R_x}{R}, \quad \theta_y = \tan^{-1}\frac{R_y}{R}, \theta_z = \tan^{-1}\frac{R_z}{R}$$

RECTANGULAR COMPONENTS IN SPACE

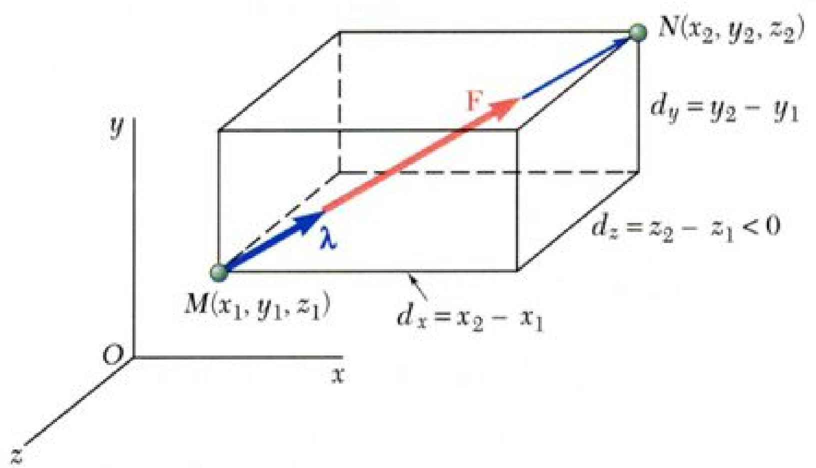
EXAMPLES

11. A tower guy wire is anchored by a means of bolt at A. The tension in the wire is 2500 N. Determine (a) the components F_x, F_y, F_z of the force acting on the bolt. (b) the angles θ_x, θ_y, and θ_z defining the direction of the force



RECTANGULAR COMPONENTS IN SPACE

Direction of the force is defined by the location of two points,
$M(x_1, y_1, z_1)$ and $N(x_2, y_2, z_2)$



$$\bar{d} = \text{vector joining } M \text{ and } N$$

$$= d_x \bar{i} + d_y \bar{j} + d_z \bar{k}$$

$$d_x = x_2 - x_1 \quad d_y = y_2 - y_1 \quad d_z = z_2 - z_1$$

$$\bar{F} = F\bar{\lambda}$$

$$\bar{\lambda} = \frac{1}{d}\left(d_x \bar{i} + d_y \bar{j} + d_z \bar{k}\right)$$

$$F_x = \frac{F d_x}{d} \quad F_y = \frac{F d_y}{d} \quad F_z = \frac{F d_z}{d}$$

RECTANGULAR COMPONENTS IN SPACE

Equilibrium of a Particle in Space

- Recall that a particle is in ***equilibrium*** when the resultant of all the forces acting on it is zero.
- Given the expression of $R_x,\ R_y$ and R_z, then

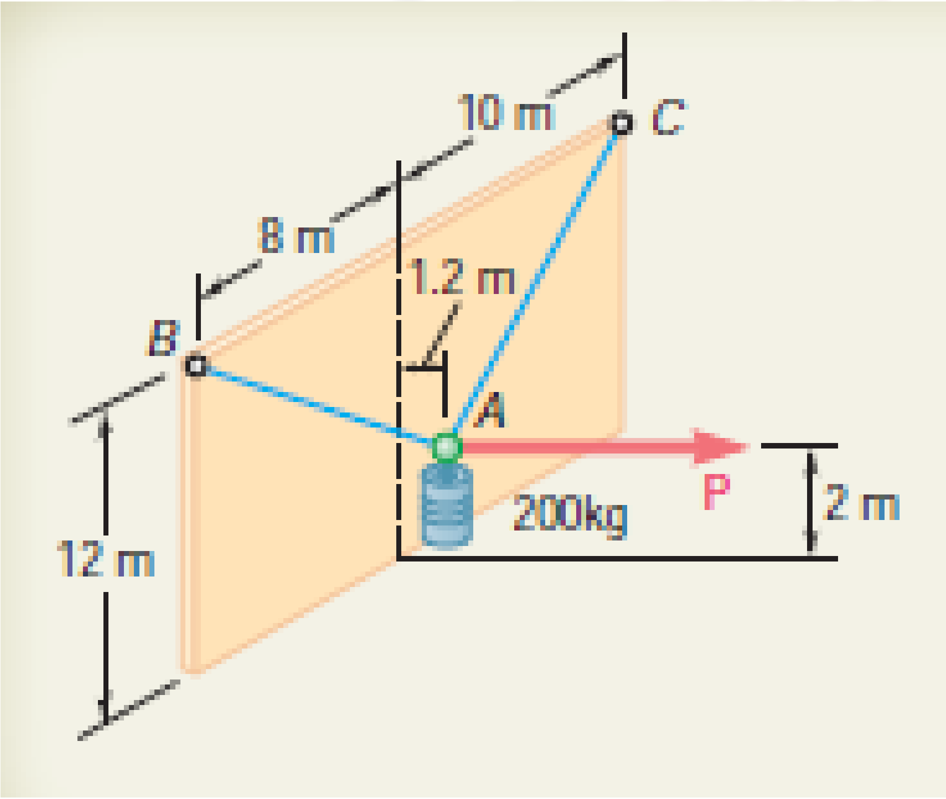
$$\sum F_x = 0, \qquad \sum F_y = 0, \qquad \sum F_z = 0$$

- The equations above establishes the conditions necessary for the equilibrium of a particle in space
- This may be used to solve problems of equilibrium of particle in space for not more than 3 unknowns
- The unknown could represent (1) the three components of a single force or (2) the magnitude of three forces each of known direction

RECTANGULAR COMPONENTS IN SPACE

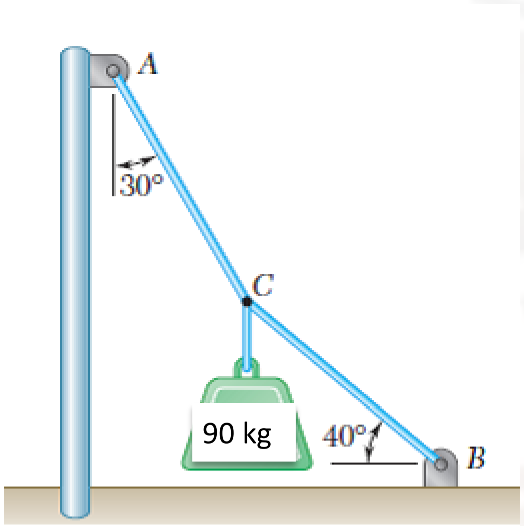
EXAMPLES

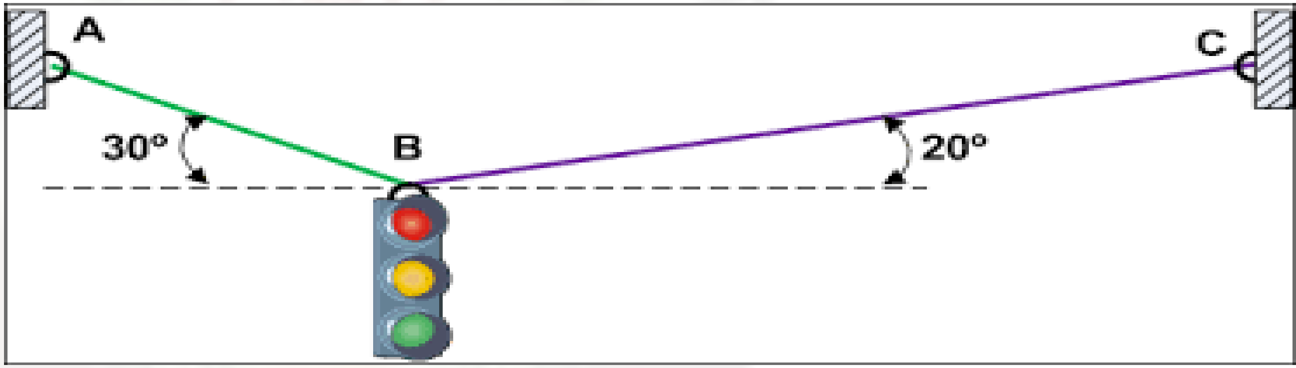
12. A 200-kg cylinder is hung by means of two cables *AB* and *AC*, which are attached to the top of a vertical wall. A horizontal force **P** perpendicular to the wall holds the cylinder in the position shown. Determine the magnitude *A* of **P** and the tension in each cable.

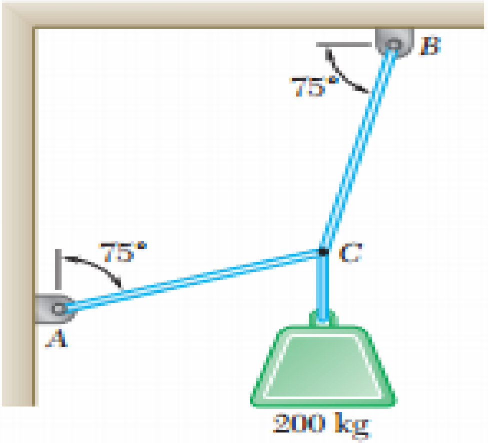


EQUILIBRIUM OF PARTICLES
MORE EXAMPLES

13. For the space diagrams represented below draw the appropriate free-body diagrams and determine the tensions in the cables as labeled. Take the weight of traffic light to be 5.5 kg



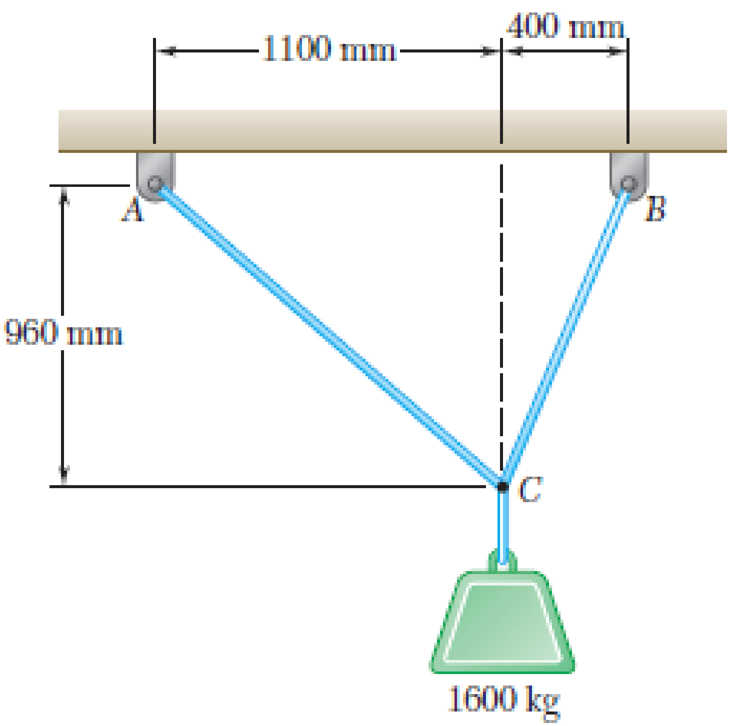


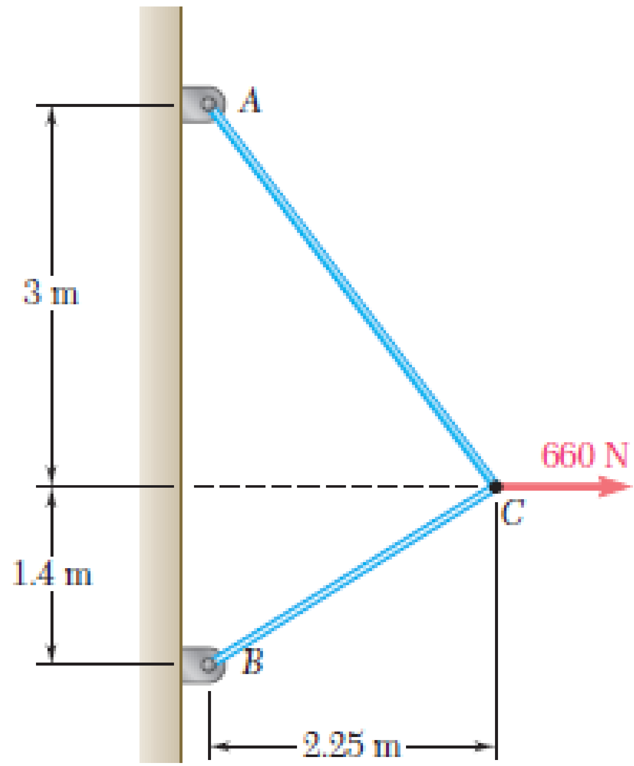


EQUILIBRIUM OF PARTICLES

MORE EXAMPLES

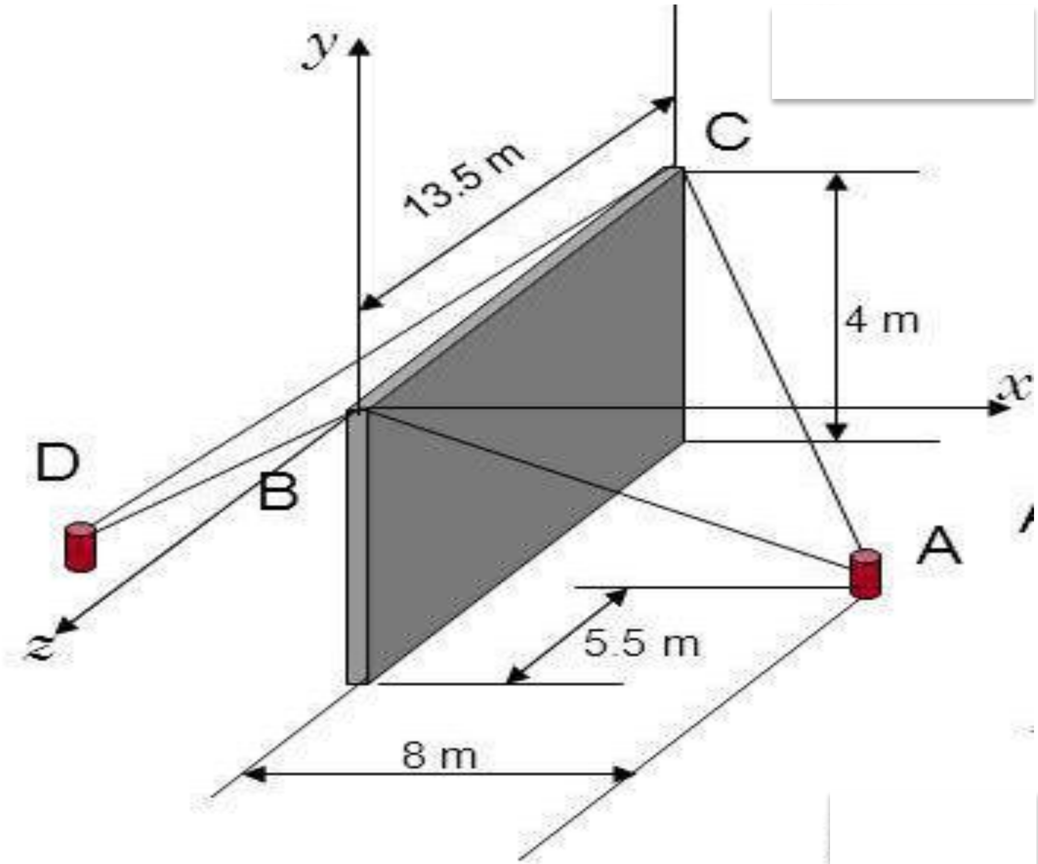
14. Two cables are tied together at C and are loaded as shown. Determine the tension (a) in cable AC, (b) in cable BC.





EXAMPLES

15. A pre-cast wall section is temporarily held by the cables shown. Knowing that tension is 4200 N in cable AB and 6000 N in cable AC, determine the magnitude and direction of the resultant of the forces exerted by cables AB and AC on stake A.



RECTANGULAR COMPONENTS IN SPACE

EXAMPLES

16. A 70-kg cylinder is supported by two cables *AC* and *BC*, which are attached to the top of vertical posts. A horizontal force **P**, perpendicular to the plane containing the posts, holds the cylinder in the position shown. Draw the free-body diagram needed to determine the magnitude of **P** and the force in each cable.